

# On Small and Large Shocks in Small and Large Banks: A (Size-dependent) Analysis of Systemic Contagion\*

Marina Balboa<sup>a</sup>, Germán López-Espinosa<sup>b</sup>, Antonio Moreno<sup>b</sup>, and Antonio Rubia<sup>a</sup>

<sup>a</sup> University of Alicante, Spain

<sup>b</sup> University of Navarra, Spain

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## Abstract

This paper analyzes how systemic contagion occurs empirically by documenting heterogeneous patterns in the intensity of systemic contagion as a function of the size of the triggering shock and as function of SIFI-related characteristics of the bank involved. Building on quantile-regression estimation, we document nonlinear patterns that configure the sensibility of the system to these shocks.

**Keywords:** Systemic risk; tail-risk dependence; proportionality; CoVaR.

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# 1 Introduction

The stability of the financial sector is particularly vulnerable to large-scale financial institutions because of the active role played by these firms in the wholesale payment and security settlement system and the outstanding size of their liabilities. As dramatically exemplified by the collapse of Lehman Brothers in 2007, large shocks transmitted by big banks can be propagated rapidly across the financial system and reach the real sector at a global scale. This has motivated a new international regulatory framework for the so-called Systemically-Important Financial Institutions (SIFIs), requiring capital surcharges, additional liquidity buffers, and tighter supervision. More generally, during periods of market turmoil and economic distress in which financial assets are prone to move together (King and Wadhvani, 1990; Ang *et al.* 2006), even shocks originated in small- and medium-sized banks can initiate spirals of systemic contagion. This raises the important issue of determining how vulnerable the financial system is to shocks differing in intensity and possibly transmitted by banks exhibiting different representative characteristics. In spite of the growing literature on systemic-risk modeling, the key question of how big an individual loss needs to be in order to spill over the whole system, given the nature of the transmitting bank, has received little attention.

The main aim of this paper is to provide greater understanding on systemic contagion by documenting heterogeneous patterns in the intensity of tail-comovements as a function of the size of the triggering shock, a bank-specific latent function that we shall refer to as Marginal Response Profile (MRP) in the sequel. Because the MRP is highly idiosyncratic and varies on a bank-to-banks basis, we characterize the average shape of this function attending to different SIFI-related characteristics, such as size, short-term wholesale funding, and volume of off-balance sheet items. Systemic contagion is captured in our approach by a significant comovement between the left tail of the conditional distribution of the returns of the system and a contemporaneous balance-sheet contraction in an individual bank. This phenomenon occurs when reductions in the market value of the assets held by the individual bank are transmitted into the overall system, for instance, through the fire-sales channel in a characteristic context of strong asset commonality; see, among others, Brunnermeier and Pedersen (2009) and Greenwood *et al.* (2015). The expected size of tail co-movements depends mainly on the marginal sensibility of the system to a specific bank and the magnitude of the triggering shock.<sup>1</sup> Naturally, the whole system exhibits different sensibilities to banks with different characteristics which, furthermore, need not necessarily be independent of the intensity of the triggering shock. As a result, systemic contagion could be featured by a nonlinear MRP, such that the expected impact of a shock could be endogenously amplified as a function of its relative size.

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<sup>1</sup>Triggering shocks can be either exogenous (e.g., an idiosyncratic event such as the failure of a financial institution), or endogenous (such as a macroeconomic imbalance); see ECB (2009) for further details.

In this paper, we characterize empirically the MRP that features the contribution of individual banks to the total risk of the system in the U.S. banking sector. Adrian and Brunnermeier (2016) proposed an econometric framework for estimating time-varying measures of systemic risk that can be conveniently adapted for this purpose. The so-called CoVaR approach builds on quantile-regression estimates of a model that relates linearly the returns of the system to the returns of an individual bank. This model assumes implicitly a constant MRP in which a shock in the left tail of an individual bank generates the same characteristic response in the left tail of the system, independently of its magnitude. This restrictive assumption can be relaxed to allow individual shocks to non-linearly feed into the left tail of the system as a function of their relative size. In our approach, we consider a piecewise-threshold-linear decomposition that splits exogenously the empirical distribution of the standardized returns of individual banks into different buckets or statistical classes characterized by the quintiles of signed returns. This allows us to appraise heterogeneous patterns characterizing the MRP of the system and make comparisons on a standardized basis. Furthermore, the empirical suitability of the model can be tested against meaningful alternatives.

We estimate the shape of the system's MRP building on the both bank holding companies and commercial banks which are publicly traded in the U.S. stock market over the period January 1990 through December 2014. We characterize MRP empirically at the 5% shortfall probability level using quantile regression and controlling for different market-wide environmental conditions. The main evidence from this analysis reveals fairly heterogeneous response patterns across the size of the triggering shocks, and across size- and other SIFI-related characteristics of the bank transmitting the shock. Consistent with previous evidence, the banking system is much more vulnerable to shocks in large banks, but our study complements this well-known fact by uncovering a number of additional features. In particular, for the class of large-scale banks in the top size-sorted quartile, the cross-sectional average MRP of the system in an adverse scenario exhibits a strong sensitivity to losses which is surprisingly stable. Accordingly, the banking system is fairly vulnerable to large banks and even relative small balance-sheet contractions in these banks can feed into the system. Additionally, whereas the cross-sectional variability of the sensitivity coefficient estimates is rather low for most extreme losses, the marginal response against small losses is much more noisy and, hence, subject to greater uncertainty. The overall evidence largely justifies the need for tighter supervision of complex institutions. In contrast, for the class of small and medium-sized banks in the bottom quartile, the overall system shows a considerable degree of resilience and only large relative movements are able to feed into the left tail of the system. The coefficients that characterize the MRP of the system against shocks to small banks are, in any case, considerably smaller.

This evidence is relevant from a regulatory perspective for at least two reasons. First, it provides more precise insight on the systemic interrelations that link losses in individual banks

to losses in the overall financial system, revealing the empirical predominance of nonlinear responses. This has clear implications, for instance, for systemic-risk measurement, since extant models, such as Adrian and Brunnermeir’s, assumes constant coefficients. Second, financial regulators set rules that must apply on banks with fairly heterogeneous characteristics. Those rules often meet proportionality criteria, allowing smaller banks and other financial institutions with a low-risk macro-prudential profile to waive the requirements that may pose unjustified burdens for them. For instance, in the US, the 2010 Dodd-Frank Act set a \$50 billion threshold in assets above which any bank automatically qualifies for Federal Reserve supervision and special regulation which demands tougher capital conditions, higher liquidity, and specific requirements such as participating in the annual stress tests. Currently, there is an intense debate on the suitability of this limit and several initiatives to raise this bar to \$500 billion. The evidence in this paper formally supports the eligibility of small and even medium-sized firms for regulatory exemptions under the principle of proportionality, since, in contrast to large banks, only large shocks seem to imply a risk of contagion. The specific analysis on the banks with median assets in the range \$50-500 billion in our sample gives little support to generally claim that these banks do not present systemic concerns.

The remaining of the paper is organized as follows. Section 2 discusses the technical aspects related to the piecewise extension of the CoVaR model in Adrian and Brunermeir (2016). Section 3 presents the data used in the paper. Section 4 discusses the empirical analysis. Finally, Section 5 summarizes and concludes.

## 2 Characterizing marginal response profiles: A CoVaR-based approach

For a certain shortfall probability  $\tau$ , consider the quantile-regression (QR) model proposed by Adrian and Brunnermeier (2016) (AB henceforth) to address the systemic contribution of the  $i$ -th individual bank to the total system:

$$X_{t,S_i} = \alpha_i(\tau) + Z'_{t-1}\gamma_i(\tau) + \delta_i(\tau) X_{t,i} + u_{t\tau}^i, \quad t = 2, \dots, T \quad (1)$$

where we use  $X_{t,S_i}$  to denote the return of the system,  $Z_t$  is a  $p$ -vector vector of state variables,  $X_{t,i}$  is the return of an individual bank,  $u_{t\tau}^i$  is a noise term obeying general standard assumptions in the QR setting, and  $(\alpha_i(\theta), \gamma_i'(\tau), \delta_i(\tau))'$  is a vector of unknown, bank-specific parameters. In a regulatory benchmark concerned with systemic contagion, it is convenient to define returns as the (relative) change of the market value of the assets portfolio held by the total system or the individual banks, so we shall understand that returns are computed in this way in the sequel. In the AB setting, the main purpose of (1) is to determine a time-varying measure

of systemic risk, termed  $\Delta\text{CoVaR}$ , which captures the contribution of the individual bank to the total risk of the system in a stressed scenario. In particular, this measure is computed as  $\Delta\text{CoVaR}(\tau) = \widehat{\delta}_i(\tau) \times \left[ \widetilde{\text{VaR}}_{t,i}(\tau) - \widetilde{\text{VaR}}_{t,i}(0.50) \right]$ , where  $\widehat{\delta}_i(\tau)$  is the quantile-regression estimate of  $\delta_i(\tau)$  in (1), and  $\widetilde{\text{VaR}}_{t,i}(\cdot)$  denotes exogenous estimates of the VaR process at a given probability level.

Our interest in this econometric framework is not motivated by the final  $\Delta\text{CoVaR}$  measure itself, but in the fact that (1) relates the conditional distribution of the returns of the system to the returns of an individual bank in a simple and fairly tractable way. In this representation,  $\delta_i(\tau)$  captures the marginal sensitivity or response of the system to a shock in the market value of the assets of an individual bank after controlling for the market-wide effects in  $Z_t$ , such that systemic contagion occurs if  $\delta_i(\tau) \neq 0$ . As discussed by López-Espinosa *et al.* (2015), equation (1) can result excessively restrictive for practical purposes because  $\delta_i(\tau)$  is assumed to be constant independently of the magnitude of the triggering shock. Consistent with previous literature in volatility and downside-risk modelling (e.g., Engle and Manganelli 2004), López-Espinosa *et al.* (2015) report empirical evidence that losses associated to negative returns trigger larger responses in the system than positive returns do. More generally,  $X_{t,S_i}$  may exhibit nonlinear marginal responses as a function of a number of latent factors. A simple and appealing way to model such dependences is to allow  $\delta_i(\tau)$  to vary on the values exhibited by the bank-specific return  $X_{t,i}$ . We discuss a generalization of the AB setting in this spirit in the remaining of this section.

First, in order to enable sound comparisons across different banks and for different categories of shocks, it is convenient to define the standardized returns  $X_{t,i}^* := (X_{t,i} - \mu_i) / \sigma_i$ , with  $\mu_i$  and  $\sigma_i$  denoting the mean and standard deviation of  $X_{t,i}$ , respectively.<sup>2</sup> Note that, for high-frequency returns,  $\mu_i$  is typically close to zero, but  $\sigma_i$  is sizeable, so this operation essentially leads to a re-scaling of the original series. Let  $\mathcal{S}_i$  be the support of  $X_{t,i}^*$ , and for a fixed  $k \geq 1$ , consider a sequence of negative thresholds  $\{\kappa_{i,j}^-\}_{j=1}^k$  partitioning the negative space of  $\mathcal{S}_i$  into  $k+1$  disjoint segments  $\{\mathcal{C}_{i,j}^-\}_{j=1}^{k+1}$ , thereby defining the variables  $X_{t,ij}^- = \{X_{t,i}^* : \kappa_{j-1,i}^- < X_{t,i}^* \leq \kappa_{j,i}^-\}$  with  $\kappa_{i,0}^- := \min \mathcal{S}_i$  and  $\kappa_{i,k+1}^- := 0$ , i.e., the set of negative observations in each of these partitions. Analogously, consider a set of positive thresholds  $\{\kappa_{i,j}^+\}_{j=1}^k$  partitioning the positive region of  $\mathcal{S}_i$  into  $k+1$  disjoint segments,  $\{\mathcal{C}_{i,j}^+\}_{j=1}^{k+1}$ , and define  $X_{t,ij}^+ = \{X_{t,i}^* : \kappa_{j-1,i}^+ < X_{t,i}^* \leq \kappa_{j,i}^+\}$  with  $\kappa_{i,0}^+ := 0$  and  $\kappa_{i,k+1}^+ := \max \mathcal{S}_i$ . Then, we can extend (1) to accommodate possible nonlinear responses against shocks in a particular bank using the piecewise-threshold-linear quantile-

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<sup>2</sup>The standardization seeks to remove effects associated to the volatility of the distribution of returns, which perhaps provides a more rigorous analysis than that based on raw returns. Nevertheless, it should be noted that the analysis on the series did not lead to different qualitative conclusions.

regression model:

$$X_{t,S_i} = \alpha_i(\tau) + Z'_{t-1}\gamma_i(\tau) + \sum_{j=1}^{k+1} \delta_{ij}^-(\tau) X_{t,ij}^- + \sum_{j=1}^{k+1} \delta_{ij}^+(\tau) X_{t,ij}^+ + u_{t\tau}^i \quad (2)$$

We then define the Marginal Response Profile (MRP) of the system at the  $\tau$ -th quantile against (standardized) shocks to the market-valued total assets held by  $i$ -th bank as the sequence of sensibility coefficients associated to the returns in each one of these categories, namely:

$$MRP_i(\tau) := (\delta_{i1}^-(\tau), \dots, \delta_{ik+1}^-(\tau), \delta_{i1}^+(\tau), \dots, \delta_{ik+1}^+(\tau))' \quad (3)$$

Some comments follow. Model (2) is a straightforward generalization of (1) in which the system is allowed to exhibit different responses to individual shocks attending to both its sign and its magnitude, after controlling for the influence of market-wide conditions captured by suitable state variables in  $Z_t$ . Model (1) in AB is nested as a particular case under the restriction  $\delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = \delta_i(\tau)$  for all  $j = 1, \dots, k+1$ , which implies  $MRP_i(\tau) = \delta_i(\tau)$ . Similarly, the asymmetric CoVaR model in López-Espinosa *et al.* (2015) arises as a particular case under the restrictions  $\delta_{ij}^-(\tau) = \delta_i^-(\tau)$  and  $\delta_{ij}^+(\tau) = \delta_i^+(\tau)$ , such that only the sign, but not the magnitude of the shock, determines the sensibility of the system. In this case,  $MRP_i(\tau)$  is a piecewise constant function exhibiting a single discontinuity at the origin. Because both models arise from imposing linear restrictions in (2), their empirical suitability can be formally tested by means of standard Wald-type tests.

The number of thresholds that characterize this model,  $k$ , could be allowed to grow such that in the limit  $MRP_i(\tau)$  would be a continuous function. We note that such approach is feasible and could be estimated using non-parametric quantile regression methods, but we do not pursue that direction in this paper. Instead, we consider a fixed, finite  $k$ , such that the resulting  $MRP_i(\tau)$  can be seen as a discrete piecewise-threshold-linear approximation of the underlying function. This approach corresponds with so-called threshold models, widely known in the literature of time-series in econometrics. The advantage of this modelling approach is that it allows to conduct inference on the coefficients and test meaningful restrictions in a fairly tractable way, as discussed previously.

The parameter vector  $\theta_i(\tau) := (\alpha_i(\tau), \gamma_i'(\tau), MRP_i'(\tau))'$  can be estimated given the sequence of threshold values by means of the standard linear quantile regression methodology. In particular, the quantile-regression estimator of  $\theta(\tau)$  given  $\Omega_{ti} := (1, Z'_{t-1,i}, X_{t,i1}^-, \dots, X_{t,ik}^+)'$ , denoted  $\widehat{\theta}_i(\tau)$ , is defined as:

$$\arg \min_{b \in \mathbb{R}^n} \sum_{t=2}^T \rho_\tau(X_{t,S} - \Omega'_{ti}b) \quad (4)$$

where  $\rho_\tau(z) = z(\tau - \mathbb{I}(z < 0))$ , with  $\mathbb{I}(\cdot)$  denoting the indicator function, and  $n = 2 \times (k+1) + p + 1$  denoting the total number of parameters to be estimated. The optimization of this objective

function involves numerical methods and, in contrast to least-squares estimation, there does not exist a closed-form solution for  $\widehat{\theta}_i(\tau)$ .

Under general conditions that do not require specifying the unknown distribution of returns,  $\widehat{\theta}_i(\tau)$  is consistent and  $T \left( \widehat{\theta}_i(\tau) - \theta_i(\tau) \right) \xrightarrow{d} \mathcal{N}(0, V_i(\tau))$  as the sample size  $T$  diverges, with  $V_i(\tau)$  denoting a finite covariance matrix that can be estimated consistently in a number of ways; see Koenker (2005). We shall implement Powell’s (1991) estimator, which combines kernel-density estimation with a heteroskedasticity-consistent covariance matrix estimation. This is the most popular way of estimating the covariance matrix in applied papers dealing with financial returns; see, among others, Engle and Manganelli (2004), Gaglianone *et al.* (2011) and Rubia and Sanchis-Marco (2013). In particular, define the outer-product matrix  $A_{T\tau} = \tau(1 - \tau) \sum_{t=2}^T \Omega_{ti} \Omega'_{ti} / T$ , and let be  $B_{T\tau} = (Th_T)^{-1} \sum_{t=2}^T \mathcal{K}(\widehat{u}_{t\tau}^i / h_T) \Omega_{ti} \Omega'_{ti}$ , where  $\mathcal{K}(\cdot)$  is a kernel function,  $h_T$  is a suitable bandwidth parameter, and  $\widehat{u}_{t\tau}^i$  denotes the estimated residuals from the quantile regression. Then, a heteroskedasticity-consistent estimate of  $V_i(\tau)$  is given by  $[B_{T\tau}]^{-1} A_{T\tau} [B_{T\tau}]^{-1}$ , which, for instance, can be used to conduct inference and construct confidence intervals for  $MRP_i(\tau)$ .

### 3 Data

We collect market and balance sheet data for both Bank Holding Companies (BHC) and Commercial Banks (CB) which are publicly traded in the U.S. stock market over the period January 1990 through December 2014, totalling 1,210 institutions. Equity market prices are obtained from Datastream on a weekly basis. Accounting data, referred to total assets, book-valued equity, long-term liabilities, as well as other well-known SIFI-related proxy variables, such as short-term wholesale funding and off-balance sheet items, are obtained from the Federal Reserve Bank of Chicago Bank Regulatory Database on a quarterly basis.

As in AB, we compute weekly market-valued total assets defined as  $A_{it} = ME_{it} \times L_{it}$  for each bank in the sample, where  $ME_{it}$  is the market value of equity, and  $L_{it}$  is the leverage ratio, defined as the total assets to book equity ratio. We then determine bank-individual returns  $X_{it}$  as the simple growth rate of  $A_{it}$ , namely,  $X_{it} = (A_{it} - A_{it-1}) / A_{it-1}$ , noting that these series capture (relative) changes in the asset portfolio of each individual bank. To circumvent the sampling frequency mismatch between market data available on a weekly basis and balance sheet disclosed on a quarterly basis, we smooth weekly the quarterly leverage ratio  $L_{it}$  using cubic spline interpolation. Results are insensitive to this consideration and, for instance, linear interpolation leads to similar qualitative evidence.

In order to implement the quantile regression methodology, we follow López-Espinosa *et al.* (2015) and require banks to be traded over at least 500 weeks on the stock market. This choice seeks to obtain a good compromise between the number of time-series observations that ensure

valid inference in the quantile-regression analysis and the total number of firms included in the filtered sample that ensures a meaningful cross-sectional analysis. The final sample is composed of 422 banks (301 BHC and 121 CB) with an average time-series length of 855 observations (maximum length over the period is 1303 observations). Table 1 reports usual descriptive statistics on the banks included in the final sample attending to bank-specific characteristics such as total assets, short-term wholesale funding, liabilities or total deposits, given the total sample, and the subsamples of BHC and CB. These statistics reveals the sheer heterogeneity between large and small companies.

**[Insert Table 1 around here]**

For the estimation of (2), and following López-Espinosa *et al.* (2012, 2015), the returns of the overall system,  $X_{t,S_i}$ , are constructed as a value-weighted average of the individual returns  $X_{it}$  after excluding the return of the  $i$ -th bank under analysis; see also Adrian and Brunnermeier (2016). More formally,  $X_{t,S_i} = \sum_{j=1}^N \omega_{t,j}^i X_{t,j}$ , with  $\omega_{t,j}^i = 0$  if  $j = i$  and  $\omega_{t,j}^i = A_{t-1,j} / \sum_{s=1, s \neq i}^N A_{t-1,s}$  otherwise, with  $N$  denoting the number of individual banks analyzed. In this approach, the proxy of the ‘system’ varies on a bank-to-bank basis as it represents the set of banks that surrounds a particular bank. In our view, this is more convenient than merely value-weighting all available individual returns for two main reasons. First, this characterization matches more naturally the idea of ‘system’ when addressing systemic contagion, since it considers a potentially contaminating agent on the one side (the individual banks), and the rest of the population on the other. Secondly, and more importantly from a methodological perspective, the estimation of the sensitivities that characterize the MRP of the system are more rigorously determined. Excluding explicitly the bank under analysis necessarily rules out the possibility of spurious tail-interrelations that otherwise may be caused by the inclusion of the same individual in both sides of equation (2). This consideration is particularly relevant when  $N$  is small, or when the relative weight of a bank in the system is particularly sizeable, as it is the characteristic case of large-capitalization banks.

Finally, following Adrian and Brunnermeier (2016), the state variables used to control for market-wide environmental conditions in the  $Z_t$  vector in (2) are the Volatility Index of the Chicago Board Options Exchange (VIX); the change in the U.S. Treasury bill secondary market 3-month rate ( $\Delta T$ -bill); the yield spread between the U.S. Treasury benchmark 10-year bonds and the U.S. 3-month T-bill (Yield Slope); the change in the credit spread between the 10-year Moody’s seasoned Baa corporate bond and the 10-year U.S. (Default Premium). Treasury bond; and the return of the S&P 500 Composite Index (Market Return). These variables have been obtained from the Federal Reserve Board’s H.15 databases. Although all these variables are strongly tied to the economic cycle and can track the time-varying dynamics of expected returns, we additionally control for shifts in the unconditional mean of the conditional quantile



process through crisis-related dummy variables as in López-Espinosa *et al.* (2015). Thus, we define an Economic Recession dummy (NBER Recessions) variable taking the value equal to one in the periods officially identified as macroeconomic recessions by the NBER (July 1990-March 1991, March 2001- November 2001 and December 2007-June 2009) and zero otherwise. In addition, and since the 2007-09 recession was a major global financial crisis, we define a specific indicator (Financial Recession) taking the value equal to one in the period from August 2007 through March 2009. The choice of this specific period is sensibly motivated by the timing of maximum disruption in money markets caused by financial uncertainty and counterparty credit risk; see also López-Espinosa *et al.* (2015) for a discussion. Table 2 reports summary statistics of these variables.

[Insert Table 2 around here]

## 4 Empirical analysis

Given the pairs  $(X_{t,i}, X_{t,S_i})'$ ,  $i = 1, \dots, N$ , and the vector of state variables  $Z_t$ , we estimate the piecewise-linear-threshold model (2) at the shortfall probability  $\tau = 0.05$  given  $k = 4$  thresholds of negative and positive returns which are given by the quintiles of the empirical distribution of the signed returns  $X_{t,i}^* \times \mathbb{I}(X_{t,i}^* < 0)$  and  $X_{t,i}^* \times \mathbb{I}(X_{t,i}^* \geq 0)$ , respectively. For the discussion that follows, recall that the parameters  $\{\delta_j^-(\tau)\}_{j \geq 1}$  and  $\{\delta_j^+(\tau)\}_{j \geq 1}$  are related to increasing classes of standardized returns such that  $X_{t,1}^-$  ( $X_{t,1}^+$ ) is formed by the smallest negative (positive) returns in the bottom quintile of the distribution of the signed standardized return series, while  $X_{t,5}^-$  ( $X_{t,5}^+$ ) includes the largest negative (positive) returns in the top quintile. Hence, the sensitivity to most extreme observations in the tails of  $X_{t,i}$  are captured by  $\delta_1^-(\tau)$  and  $\delta_5^+(\tau)$ , while  $\delta_5^-(\tau)$  and  $\delta_1^+(\tau)$  capture the marginal response of the system against mild departures from zero.

The choice of  $k$  and  $\tau$  seeks a fair balance between the number of observations included in each partition, resulting from the total number of classes considered, and the precision in the QR estimation. Ideally, a large number of classes would produce a continuous MRP function. In a finite sample, however, an increasing partitioning reduces the number of observations within each class, which compromises the accuracy in linear QR estimation, particularly, at extreme quantiles; see Chernozhukov (2005) and Koenker (2005). In this context, the estimation of parameters related to dummy variables in the QR setting can be very imprecise at extreme quantiles and, in any case,  $\tau = 0.05$  is a usual choice in the downside-risk literature.<sup>3</sup> Given

<sup>3</sup>The Basel framework requires the 1% ( $\tau = 0.01$ ) to determine regulatory capital adequacy, but higher quantiles are also applied for different purposes. Publicly traded firms are required to disclose quantitative market risk measures in their financial statements under SEC rules, being VaR one out of three possible disclosing formats entitled. The SEC rule, effective since June 1998, states a 5% VaR or lower risk level, but

the QR estimation of model (2), the asymptotic covariance matrix is inferred using Powell’s estimator, as described previously, using a Gaussian kernel and a bandwidth parameter  $h_T$  selected according to the rule  $h_T = 0.9 \times \min \{\hat{\sigma}_u, IQR_{\hat{u}}\} \times T^{-1/5}$ , where  $\hat{\sigma}_u$  and  $IQR_{\hat{u}}$  denote the sample standard deviation and the sample interquartile range of  $\hat{u}_{\tau t}^i$ .

## 4.1 Main evidence

Owing to the large number of banks in our analysis, we present and discuss summarizing results referring to the whole sample and to a conditional analysis that groups banks according to their degree of systemic importance. While there is no formal definition of systemic importance, this concept can be related to multiple firm-specific dimensions. In order to ensure that results are not driven by the specific choice of a proxy variable, we consider alternative indicators related to systemic importance given publicly available data. In our analysis, these are determined by the time-series medians of total assets (TA), short-term wholesale funding (STWSF), and off-balance sheet items (OBSI). Results based on other related variables (e.g., long-term liabilities), did not lead to different conclusions and are omitted, but available upon request. Tables 3, 4 and 5 report the medians of the parameter estimates of model (2) over the total sample and given groups of banks given by the quartiles of TS, STWSF, and OBSI. In addition, these tables report the frequencies of rejection of the  $t$ -statistics for individual significance of the parameter estimates at the usual 95% confidence level over the total and conditional samples.

**[Insert Table 3 around here]**

Since the evidence that emerges from different SIFI-related variables is completely similar, for ease of exposition we focus on the results for TA, reported in Table 3. The empirical quartiles of this variable define a Top category formed by banks with TA greater than \$4.6 billion; the third quartile (Q3) is formed by banks with TA between \$1.4 and \$4.6 billion; the second quartile (Q2) is formed by banks with TA between \$0.7 \$1.4 billion; finally, the Bottom category is formed by banks with TA smaller than \$0.7 billion. From a systemic-risk perspective, the first category poses the major interest as it includes the largest banks in the system. In the final sample analyzed in this paper, this category includes 19 banks with median values of TA greater than \$50 billion and, consequently, eligible for tighter Federal Reserve supervision under the 2010 Dodd-Frank Act. Among the largest banks in this category, six BHC have TA greater than \$500 billion, and eight BHC have been considered as global systemically important bank (G-SIB) in a total list of 30 banks (as by November 2014) by the Financial Stability Board using the assessment methodology developed by the Basel Committee on Banking Supervision.

We first discuss the estimates that characterize the average shape of the MRP of the system given all the banks in the sample; see Table 3. The most noticeable feature is the abrupt 

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also permits higher rates provided economic justification.

discontinuity around the zero threshold. For negative returns in the classes  $(X_{t,1}^-, \dots, X_{t,5}^-)'$ , the coefficients that characterize the MRP show median values that range non-monotonically from  $\delta_1^-(\tau) = 0.020$  to  $\delta_5^-(\tau) = 0.027$ . In contrast, for positive returns  $(X_{t,1}^+, \dots, X_{t,5}^+)'$ , the medians of the estimated sensitivities are several orders of magnitude smaller (e.g.,  $\delta_1^+(\tau) = 0.002$  and  $\delta_5^+(\tau) = 0.003$ ), which leads to a sharp discontinuity in the average response of the system against mildly negative and slightly positive returns around zero. According to these estimates, therefore, the characteristic MRP is a positive and strongly asymmetric function such that negative shocks tend to trigger larger responses under stressed conditions.

In addition, the estimates suggest that the average MRP is fairly stable on average for either negative or positive shocks, since the differences between the coefficients in those regions are relatively small. Nevertheless, the analysis of individual significance shows that the coefficients related to more extreme shocks tend to be more significant in statistical terms, particularly, in the negative region. For negative shocks, the average frequency of rejection of  $H_0 : \delta_1^-(\tau) = 0$  at the 95% confidence level is 74.41%. This ratio decays monotonically as the magnitude of the shock decreases such that  $H_0 : \delta_5^-(\tau) = 0$  shows an overall cross-sectional rejection ratio of approximately 17%. Accordingly, extreme negative shocks would spill over the system systematically and, although there is a greater deal of uncertainty in the response to small losses, the system exhibits statistically significant responses which are similar in size to those of large losses for a sizable proportion of large banks in the sample. We shall turn back to this issue later on when analyzing the frequencies of rejections of composite tests specifically intended to check coefficient homogeneity. Finally, and in sharp contrast to negative returns, positive individual returns have a much more marginal effect on the left-tail of the system in a stressed scenario. With the exception of the largest positive shocks, there is little evidence of tail co-movement. This is not surprising because positive shocks lack of the channels that enable negative shocks to quickly spread out; see López-Espinosa *et al.* (2015) for a discussion.

**[Insert Table 4 and 5 around here]**

We now discuss the results from the conditional analysis when banks and their respective estimates are sorted attending to the size of TA. Clearly, systemic contagion does not evolve independently of this individual characteristic. Both the magnitude of the estimated coefficients that determine the SMRP and the evidence of statistical significance of these coefficients are strongly affected by bank's size. Broadly speaking, the SMRP against shocks in large banks, particularly those belonging to the Top category, is pushed upward in relation to the unconditional SMRP. This is consistent with a higher vulnerability and leads to stronger intensity in systemic contagion across all the categories of shocks analyzed. Similarly, the rejection ratios of the tests of individual significance on the related coefficients largely increase, particularly, for banks in extreme quartiles. For example, for the Top size-sorted quartile, the

(conditional) cross-sectional median estimate of  $\delta_1^-(\tau)$  is 0.028, and the rejection frequency of  $H_0 : \delta_1^-(\tau) = 0$  increases to 91.43%. According to our estimates, the largest sensitivity coefficients and the largest rejection frequencies of the individual  $t$ -statistics are consistently related to the biggest banks in the sample. On the other hand, the group of smallest banks exhibit a median SMRP formed by coefficients closer to zero and hardly significant in many cases. For instance, the median of the estimated sensitivities  $\delta_1^-(\tau)$  of the system against the most extreme negative shocks to small banks in  $X_{t,1}^-$  is 0.012, which is statistically significant only in 49.53% of the banks analyzed. To put this into perspective, note that the overall system is much more sensitive to smallest losses in the largest banks than to the most extreme losses in the category of the smallest banks, noting that returns have been standardized to make such comparison sound and meaningful. Accordingly, therefore, small and medium-sized banks are, in general, not a concern from a systemic perspective.

**[Insert Figures 1 to 2 around here]**

In order to provide further insight on the pattern of systemic contagion, Figure 1 shows the average shape of MRP against banks belonging to the Top and Bottom size-sorted quartiles according to the estimates reported in Table 3. Medians of parameter coefficient estimates are displayed together with bootstrap-based 95% confidence-intervals; see Chernick (2008). For completeness, Figure 2 shows the SMRP and bootstrapped confidence intervals against banks belonging to the Q3 and Q2 size-sorted categories. When comparing both figures, it is clear that there is a smooth transition from the characteristic pattern exhibited by the MRP in top-sized banks and that related to bottom-sized banks. Consequently, we directly comment on the main evidence in Figure 1, for which differences caused by size are magnified.

All the empirical features discussed previously can be recognized immediately in Figure 1, such as the strong asymmetric pattern around the zero return (which is particularly evident in the top-size group), and the homogeneous patterns that sensibility coefficients seem to exhibit on average against either negative or positive shocks. For the biggest banks in the sample, the medians of the coefficients that characterize the average MRP against negative shocks are significant and positive at the 95% confidence level. The  $X_{t,5}^-$  class, composed by mild losses, has attached a bootstrapped confidence interval with the largest amplitude (i.e., higher parameter uncertainty) in our sample. In contrast, the largest losses belonging the  $X_{t,1}^-$  class have attached median estimates with very low dispersion, suggesting that extreme losses are consistently associated with larger marginal responses in the system. This heterogeneous pattern is likely due to the fact that extreme individual losses tend to systematically occur during periods of market distress, whereas small losses can occur during either calmed or stressed periods for which the system would exhibit different sensitivities. Similarly, the coefficients that feature the MRP against positive shocks are mostly significant, except for small gains in the  $X_{t,1}^+$

quintile, but all of them are close to zero.

In shap contrast, for the set of smallest banks, the MRP is only positive and statistically significant against large negative shocks in the  $X_{t,1}^-$ ,  $X_{t,2}^-$  and  $X_{t,3}^-$  classes, meaning that only large negative shocks are able to trigger, on average, a significant response under stressed conditions. In any case, the coefficients that feature these responses are considerably smaller than those that characterize the MRP against big banks. Finally, whereas we have discussed results focusing on size as proxied by TA, we stress that the evidence is fairly robust against the choice of the indicator of systemic importance and similar results arise under alternative variables. Figure 3 characterizes the average shape of the MRP given banks that belong to top and bottom STWSF-sorted quartiles. The empirical patterns that emerge match closely those discussed previously.

[Insert Figure 3 around here]

## 4.2 Testing composite hypotheses on the SMRP shape

We now turn our attention to formally determine whether there exists sufficient regularity on the coefficient estimates that characterize the shape of the SMRP as to, for instance, accept constant responses for different categories of shocks. To this end, Table 6 reports the frequencies of rejection at the 95% confidence level of different composite hypotheses involving sets of parameters, given the total sample, and given the quartiles of the sorting variables TA, STWSF and OBSI. In particular, we address the null hypothesis that system is not affected by individual shocks after controlling for market-wide effects, namely,  $H_{0,Ind} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = 0$  for  $j = 1, \dots, 5$ . This is a formal test for the existence of tail-comovements and systemic contagion which assumes  $SMRP_i(\tau) = 0$ , i.e., the tail of the system is only driven by market-wide effects. Additionally, we test for the suitability of the linear restrictions that gives rise to the symmetric and asymmetric CoVaR model in AB and López-Espinosa *et al.* (2015), namely,  $H_{0,Sym} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = \delta_i(\tau)$  and  $H_{0,Asym} : \delta_{ij}^-(\tau) = \delta_i^-(\tau)$ ,  $\delta_{ij}^+(\tau) = \delta_i^+(\tau)$ , respectively. As discussed previously, the symmetric CoVaR model implies a constant SMRP, while the asymmetric CoVaR implies a piecewise constant SMRP with a single discontinuity at zero. Finally, we test for the equality of coefficients in the negative and positive regions, considering the tests  $H_{0,EqLoss} : \delta_{ij}^-(\tau) = \delta_i^-(\tau)$  and  $H_{0,EqGain} : \delta_{ij}^+(\tau) = \delta_i^+(\tau)$ . Note that  $H_{0,EqLoss}$  ( $H_{0,EqGain}$ ) restricts the coefficients related to negative (positive) returns of the SMRP to be the same, but leaves unrestricted the coefficients associated to positive (negative) returns.

[Insert Table 6 around here]

The hypothesis  $H_{0,Ind}$  of no systemic interrelations is strongly rejected, both unconditionally

and across all SIFI-related categories. While the rejection frequencies of  $H_{0,Ind}$  increase with the proxies of systemic importance and reach 100% for banks in the top quartiles of these variables, these ratios are sizable even in the bottom classes of the SIFI-related variables and are greater than 84% in all cases. This evidence underlines the fact that all banks, whether big or small, are interconnected under adverse market circumstances. It is worth recalling at this point that the return of the system is computed by explicitly excluding the bank-individual return, so this massive evidence of interrelation is not spuriously caused by a mechanical correlation effect and needs to be attributed to balance-sheet interconnection effects.

Similarly, the constant-coefficient restriction  $H_{0,Sym}$  in the linear CoVaR model is mostly rejected. Rejection frequencies are not smaller than 66%, and increase to 90% for banks with high-risk systemic profile, as proxied by any of the indicators considered. This result provides massive statistical support to the hypothesis that the SMRP of the system exhibits some form of non-linearity, particularly, in large-scale, complex banks. On the other hand, the rejection frequencies of  $H_{0,Asym}$  are sizable, but much more moderate than those of  $H_{0,Sym}$ , suggesting that adding further parametric structure to the linear model succeeds in capturing systemic interdependence more accurately. In particular, the unconditional rejection frequency of  $H_{0,Asym}$  against the more general specification analyzed here is 45.02%. There is little evidence of patterns on why this test is accepted or rejected, since this proportion remains remarkably stable when conditioning along the quartiles of the SIFI-related variables.

While imposing constant marginal responses for either negative or positive returns may not be completely accurate, the results show that this model is not unreasonable from a parsimonious perspective. In this context, the results from testing  $H_{0,EqLoss}$  and  $H_{0,EqGain}$  can provide greater insight on characteristic shape of the SMRP. The null hypothesis  $H_{0,EqLoss}$  shows the lowest rejection ratios among the different composite hypotheses analyzed. For instance, for banks in the top quartile of TA, the rejection ratio of  $H_{0,EqLoss}$  is 20%, suggesting that the system's sensibility against losses is stable for most of the large banks. Similar evidence arises under STWSF-sorting (20.95%) and OBSI-sorting (23.81%). The rejection frequencies of  $H_{0,EqLoss}$  are slightly greater for small banks (e.g., it is 34.58% for the bottom quartile of size), which is consistent with greater heterogeneity in this class of banks. This result fully agrees with the pattern reported in Figure 1; see also Table 3. In particular, the SMRB of the system for the class of small banks tends to exhibit on average positive and significant coefficients only when negative shocks are large. In contrast, for moderate shocks in the upper quintiles, the coefficients that characterize the average SMRP are not significant. Finally, the rejection ratios of  $H_{0,EqGain}$  are small, but tend to be much larger than that  $H_{0,EqLoss}$ . As in the previous case, this is nothing but the natural reflection of the general picture that emerges from Table 3 and Figure 1, since, in general terms, it is necessary to consider fairly large positive shocks to cause a co-movement in the system in a stressed scenario.

The overall picture that emerges from this analysis strongly rejects the hypothesis of a constant MRP which would be consistent with the constant-response model in Adrian and Brunnermeir (2010), particularly, for banks characterized by SIFI-related features. The model with asymmetric MRP characterized in López-Espinosa *et al.* (2015) is more accurate from a parsimonious perspective. Nevertheless, a more precise picture can generally be captured when the parameters are allowed to be freely determined across different categories of shocks.

### 4.3 Proportionality in the U.S. financial sector

The 2010 Dodd-Frank Act set a \$50 billion demarcation line in assets to identify systemically important institutions in the U.S. financial system. Firms with assets above this threshold are automatically designated for tighter Federal Reserve oversight and must comply with special requisites such as participating in the annual stress tests. The choice of this particular threshold qualifies the largest banks in the industry, but nevertheless has been deemed as arbitrary and excessively conservative for the detractors of this legislation. In May 2015, the Banking Committee of the U.S. Senate proposed a draft for setting new legislation on the sector in which the critical threshold would be raised to \$500 billion, giving the Federal Reserve the right to apply enhanced supervisory requirements to banks with assets between \$50 billion and \$500 billion. Because of its potential impact on the stability of the financial system, this initiative has found a strong opposition and it there is a fierce debate on whether the requirements of the Dodd-Frank Act should be relaxed or not.

Using the econometric methodology implemented in our analysis, we can address the sensitivity of the system to banks characterized by different sizes (as determined by the median of total assets) around these threshold levels. To this end, we determine the median MRP of the system against banks with a time-series median of TA falling in any of following categories: med(TA) larger than \$250 billion; med(TA) between \$100 and \$250 billion; med(TA) between \$50 and \$100 billion, and med(TA) smaller than \$50 billion. Table 7 below reports the main results from the estimation of the piecewise-linear-threshold model (2) at the shortfall probability  $\tau = 0.05$  with  $k = 4$  as in the previous section.

**[Insert Table 7 around here]**

The picture that emerges from this analysis broadly agrees with that discussed in the previous section, showing that shocks originated in large banks consistently spill over the system in a stressed scenario, whereas the system is significantly more resilient against shocks in small and medium-sized banks. The empirical shape of the MRP exhibited by the system against shocks to banks with assets in the region in dispute does not generally support the suitability of a different treatment given to these banks under the argument that they do not pose a threat to the system. Whereas it may be true that for certain cases that the burden of carrying out

stress tests annually and other of the tough requirements may exceed the benefits for stability of the system, and that it could be harmless to slightly raise this limit to provide further proportionality, or, alternatively, relax some of the requirements (e.g., conducting stress tests bi-annually), the evidence in this paper does not support the convenience of dramatic changes in the regulation and advises, at very least, for the convenience that the Federal Reserve retain the right to implement enhanced supervision.

## 5 Concluding remarks

Under stressed conditions, idiosyncratic shocks originated in small and medium sized-banks –which do not pose a serious concern for the integrity of the financial system in normal conditions– can spill over and affect other banks, which has motivated a considerable attention from regulator and academic researchers. Nevertheless, since the econometric modelling of systemic risk is a relatively new field, several questions remain unsolved. In this paper, we have analyzed if the marginal response that features tail comovements between the banking system and individual banks can be characterized by constant responses or exhibit size-dependent non-linear patterns. To this end, we have implemented a piecewise-linear threshold model on US banking data building on a generalization of the CoVaR setting of Adrian and Brunnermeier (2016) to explicitly accommodate heterogeneous responses. This analysis brings new empirical evidence on the vulnerability of the financial system and the suitability of constant-response models used in the literature.

For large banks, characterized by a large volume of total assets and liabilities and intense activity in short-term wholesale markets, our analysis reveals that the average marginal response function of the system is fairly sensitive even to small negative shocks, i.e., thereby underlining the relevance systemic interconnections between these firms and the whole system. For the vast majority of large-scale banks, the assumption that these systemic links are constant independently of the size of the shock turns out to be restrictive and is largely rejected in statistical terms. Consistent with López-Espinosa et al. (2015), our analysis reveals that a major consideration in the CoVaR modelling is whether shocks to individual banks are positive or negative. Whereas the tail of the loss function of the system exhibits a small sensitivity to positive shocks, even small negative shocks feed in the system. Our analysis reveals that the marginal response of the system, conditional to the sign of the shock, is remarkably stable for most banks analyzed, which generally supports the empirical suitability of the so-called asymmetric CoVaR model by López-Espinosa et al. (2015). Nevertheless, for a significant share of banks, introducing further heterogeneity in parameters leads to improvements. On the other hand, for the group of small banks in the sample typically characterized by traditional lending activities and small size, the analysis of the marginal response profile that features



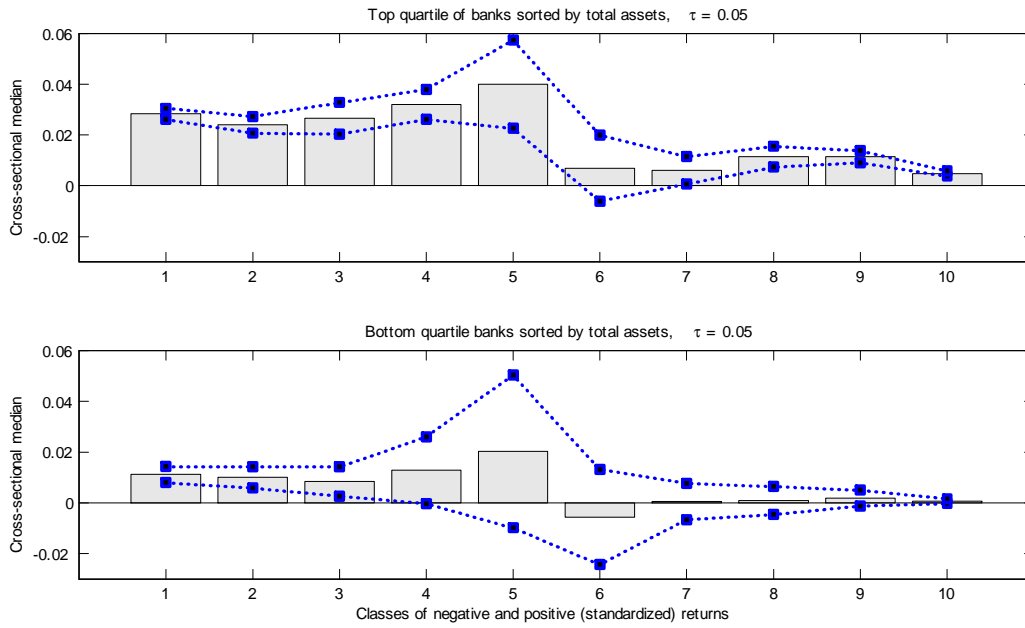
tail-comovements also reveals non-linearities, but only large negative shocks are able to affect the whole financial system.

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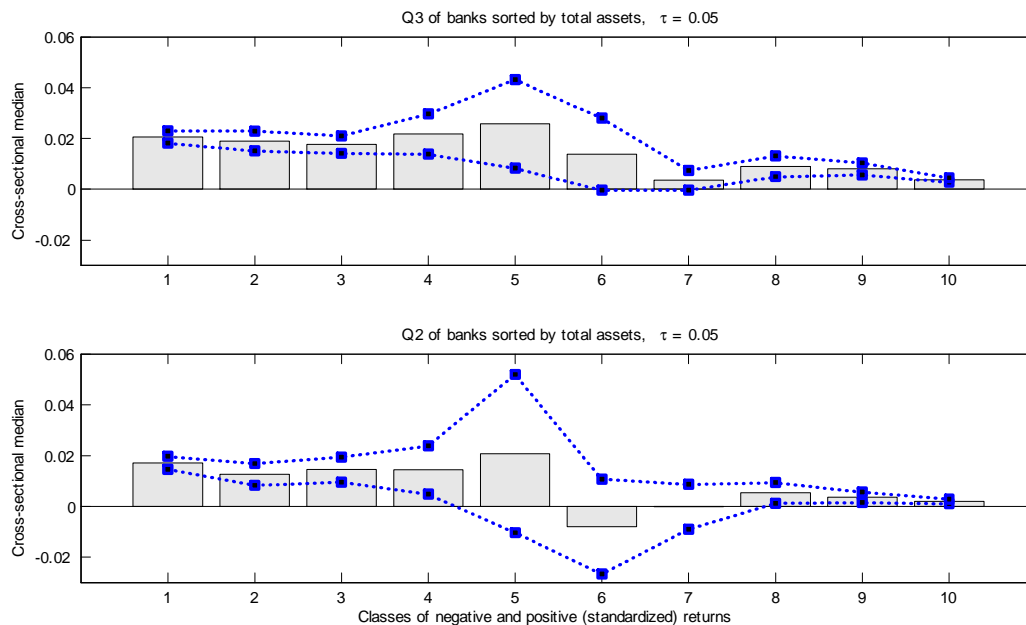
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## 6 Figures and tables

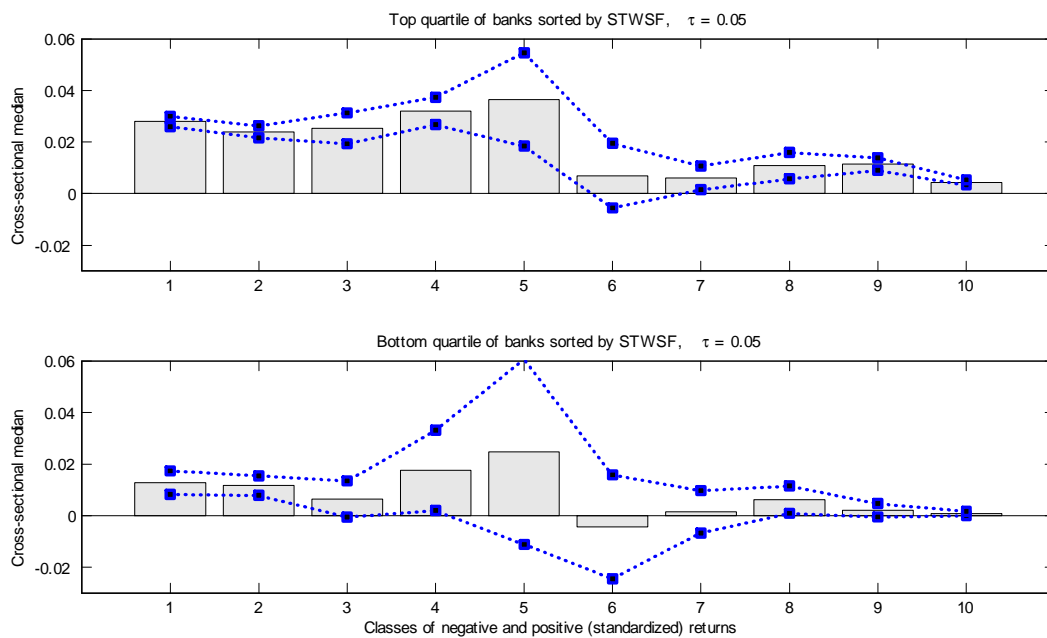
**Figure 1.** Cross-sectional medians of the parameter estimates of the coefficients  $\{\delta_i^-(\tau)\}_{j=1}^5$  and  $\{\delta_i^+(\tau)\}_{j=1}^5$  (grey bars) that characterize the MRP of the system at  $\tau = 0.05$  against standardized shocks to banks in size-sorted Top and Bottom quartiles reported in Table 3. Blue dotted-dashed lines show bootstrapped-based 95% confidence intervals for the median. The first five classes in the horizontal axis correspond to the quintiles  $X_{t,1}^-, \dots, X_{t,5}^-$  of negative returns, whereas the remaining five classes correspond to the quintiles  $X_{t,1}^+, \dots, X_{t,5}^+$  formed by positive returns.



**Figure 2.** Cross-sectional medians of the parameter estimates of the coefficients  $\{\delta_i^-(\tau)\}_{j=1}^5$  and  $\{\delta_i^+(\tau)\}_{j=1}^5$  (grey bars) that characterize the MRP of the system at  $\tau = 0.05$  against shocks to size-sorted banks in the Q3 and Q2 reported in Table 3. See caption in Figure 1 for details.



**Figure 3** Cross-sectional medians of parameter estimates of the coefficients  $\{\delta_i^-(\tau)\}_{j=1}^5$  and  $\{\delta_i^+(\tau)\}_{j=1}^5$  (grey bars) that characterize the MRP of the system at  $\tau = 0.05$  against shocks to banks sorted by short-term wholesale funding belonging to the Top and Bottom quartiles reported in Table 4. Blue dotted-dashed lines show 95% bootstrapped-based confidence intervals for the median. See caption in Figure 1 for details.



**Table 1.** Cross-sectional statistics (mean, median, standard deviation, minimum and maximum) of different bank-specific characteristics of the individuals included in the final sample. By columns, Observations represents the number of time-series observations, while the remaining columns represent the time-series medians of total assets (TA), short-term wholesale funding (STWSF), liabilities (LBT) and deposits (DEP).

	Observations	Median TA	Median STWSF	Median LBT	Median DEP
<b>All Banks</b>					
Mean	855.65	17222630.71	2837865.57	15793990.98	8545628.55
Median	789.00	1428873.50	73384.11	1312930.20	923215.95
Std	262.72	106724702.44	21731992.85	97998722.72	45146191.42
Min	501.00	46287.00	0.00	43294.56	47029.95
Max	1303.00	1821394782.00	349475300.58	1660404433.90	670227177.67
<b>BHC</b>					
Mean	863.88	16848688.25	2967431.64	15423635.86	9143222.54
Median	796.00	1537433.50	84787.42	1391262.58	1002596.79
Std	264.98	107916157.94	22525211.35	98924856.11	46852027.75
Min	501.00	238413.50	200.51	115354.77	123235.55
Max	1303.00	1821394782.00	349475300.58	1660404433.90	670227177.67
<b>CB</b>					
Mean	751.81	21939130.69	1203661.26	20465244.35	1008233.45
Median	730.00	669751.00	6870.70	543444.34	431027.38
Std	209.30	91763115.57	5354483.01	86767686.90	1617594.37
Min	507.00	46287.00	0.00	43294.56	47029.95
Max	1277.00	502525626.00	29408187.63	476661831.68	8214161.39

**Table 2.** Descriptive statistics of the economic and financial state variables used as predictive variables in the CoVaR modeling. The variables are the implied volatility index (VIX), the annualized market return of the S&P500, the change in the U.S. Treasury bill secondary market 3-month rate ( $\Delta T$ -bill), the yield spread between the U.S. Treasury benchmark bond 10-year and the U.S. 3-month T-bill (Yield Slope), and the credit spreads between the 10-year Moody's seasoned Baa corporate bond and the 10-year U.S. Treasury bond (Default Premium). The descriptive statistics show the sample mean, standard deviation, skewness, excess kurtosis, minimum, maximum, first quartile (Q1), median, third quartile (Q3), and the first-order autocorrelation parameter.

	VIX	MarketReturn	$\Delta T$ -Bill	Yield Slope	Default Premium	NBER Recession	Financial Recession
Mean	19.9472	0.0016	-0.0060	1.8417	2.3449	0.0599	0.0698
Volatility	7.9035	0.0230	0.0918	1.1720	0.7846	0.2372	0.2549
Skewness	2.0032	-0.4655	-2.1622	-0.2489	1.7282	3.7149	3.3794
Excess kurtosis	6.8085	5.5128	21.8489	-1.0614	4.8018	11.8187	9.4345
Min	9.9100	-0.1820	-1.0000	-0.7700	1.2600	0	0
Max	72.9200	0.1203	0.5900	3.8100	6.1200	1	1
Q1	14.2300	-0.0110	-0.0300	0.8300	1.7200	0	0
Median	18.1600	0.0026	0.0000	1.9600	2.1900	0	0
Q3	23.3650	0.0136	0.0300	2.8000	2.8100	0	0
Autocorrelation Coef.	0.9637	-0.0856	0.1751	0.9948	0.9965	0.9727	0.9882

**Table 3.** QR parameter estimates of model (2) at  $\tau = 0.05$ , pseudo- $\mathcal{R}^2$ , and analysis of individual significance. The first columns reports the median of the parameter estimates over all the banks in the sample and over size-sorted quartiles (Top and Bottom corresponds to banks with the largest and smallest capitalization, respectively). The second part of the table reports the rejection frequencies of the individual  $t$ -statistics at the usual 95% confidence level.

Variables	Median of estimates					Frequencies of rejection individual $t$ -test				
	Total Sample	Top	Q3	Q2	Bottom	Total Sample	Top	Q3	Q2	Bottom
Constant	0.007	-0.001	0.009	0.014	0.011	42.18%	26.67%	44.76%	51.43%	45.79%
VIX	-0.002	-0.001	-0.002	-0.002	-0.002	96.21%	92.38%	97.14%	99.05%	96.26%
Market Return	0.060	0.027	0.065	0.073	0.073	32.23%	23.81%	28.57%	33.33%	42.99%
$\Delta$ T-bill	-0.003	0.010	0.003	-0.024	-0.016	41.47%	33.33%	36.19%	47.62%	48.60%
Yield Slope	0.002	0.001	0.001	0.002	0.003	29.86%	28.57%	19.05%	29.52%	42.06%
Default Premium	-0.005	-0.001	-0.004	-0.007	-0.009	37.68%	21.90%	27.62%	44.76%	56.07%
NBER Recessions	-0.001	-0.006	0.000	0.001	0.001	22.04%	27.62%	18.10%	18.10%	24.30%
Financial Recession	-0.034	-0.030	-0.036	-0.038	-0.032	88.63%	100.00%	91.43%	91.43%	71.96%
$X_{t,1}^-$	0.020	0.028	0.021	0.017	0.012	74.41%	91.43%	80.00%	77.14%	49.53%
$X_{t,2}^-$	0.017	0.024	0.019	0.013	0.011	42.18%	65.71%	48.57%	29.52%	25.23%
$X_{t,3}^-$	0.018	0.027	0.018	0.015	0.009	28.91%	48.57%	28.57%	20.95%	17.76%
$X_{t,4}^-$	0.024	0.032	0.022	0.014	0.013	17.30%	27.62%	17.14%	13.33%	11.21%
$X_{t,5}^-$	0.027	0.040	0.026	0.021	0.020	17.30%	23.81%	11.43%	14.29%	19.63%
$X_{t,1}^+$	0.002	0.007	0.014	-0.008	-0.009	13.27%	14.29%	11.43%	14.29%	13.08%
$X_{t,2}^+$	0.003	0.006	0.004	0.000	0.000	10.66%	12.38%	9.52%	8.57%	12.15%
$X_{t,3}^+$	0.007	0.011	0.009	0.005	0.001	13.27%	18.10%	15.24%	7.62%	12.15%
$X_{t,4}^+$	0.006	0.012	0.008	0.004	0.002	21.80%	40.00%	22.86%	16.19%	8.41%
$X_{t,5}^+$	0.003	0.005	0.004	0.002	0.001	28.91%	37.14%	26.67%	23.81%	28.04%
Pseudo- $\mathcal{R}^2$	33.1%	37.8%	33.6%	31.5%	30.3%	-	-	-	-	-



**Table 4.** QR parameter estimates of model (2) on all the banks in the sample, and on quartiles sorted by short-term wholesale funding. See captions in Table 3 for details.

Variables	Median of estimates: STWSF-sorted					Frequencies of rejection individual $t$ -test				
	Total Sample	Top	Q3	Q2	Bottom	Total Sample	Top	Q3	Q2	Bottom
Constant	0.007	-0.001	0.007	0.013	0.011	42.18%	30.48%	46.67%	47.62%	43.93%
VIX	-0.002	-0.001	-0.002	-0.002	-0.002	96.21%	92.38%	95.24%	100.00%	97.20%
Market Return	0.060	0.026	0.081	0.051	0.073	32.23%	22.86%	37.14%	32.38%	36.45%
$\Delta$ T-bill	-0.003	0.012	-0.006	-0.020	-0.013	41.47%	31.43%	41.90%	48.57%	43.93%
Yield Slope	0.002	0.001	0.001	0.002	0.002	29.86%	29.52%	20.95%	33.33%	35.51%
Default Premium	-0.005	-0.001	-0.005	-0.007	-0.007	37.68%	21.90%	38.10%	46.67%	43.93%
NBER Recessions	-0.001	-0.005	-0.001	0.002	0.001	22.04%	29.52%	20.95%	18.10%	19.63%
Financial Recession	-0.034	-0.029	-0.038	-0.034	-0.033	88.63%	99.05%	91.43%	89.52%	74.77%
$X_{t,1}^-$	0.020	0.028	0.021	0.016	0.013	74.41%	89.52%	79.05%	72.38%	57.01%
$X_{t,2}^-$	0.017	0.024	0.018	0.014	0.012	42.18%	63.81%	48.57%	32.38%	24.30%
$X_{t,3}^-$	0.018	0.025	0.017	0.017	0.006	28.91%	46.67%	25.71%	25.71%	17.76%
$X_{t,4}^-$	0.024	0.032	0.019	0.017	0.018	17.30%	26.67%	17.14%	9.52%	15.89%
$X_{t,5}^-$	0.027	0.037	0.024	0.021	0.027	17.30%	21.90%	11.43%	18.10%	17.76%
$X_{t,1}^+$	0.002	0.007	0.009	-0.003	-0.006	13.27%	14.29%	15.24%	10.48%	13.08%
$X_{t,2}^+$	0.003	0.006	0.003	0.001	0.002	10.66%	10.48%	10.48%	6.67%	14.95%
$X_{t,3}^+$	0.007	0.011	0.008	0.005	0.006	13.27%	18.10%	14.29%	8.57%	12.15%
$X_{t,4}^+$	0.006	0.012	0.006	0.005	0.002	21.80%	38.10%	20.95%	14.29%	14.02%
$X_{t,5}^+$	0.003	0.004	0.004	0.002	0.001	28.91%	36.19%	30.48%	24.76%	24.30%
Pseudo- $\mathcal{R}^2$	33.1%	37.6%	33.5%	31.8%	29.8%	-	-	-	-	-

**Table 5.** QR Parameter estimates of model (2) on all the banks in the sample, and on quartiles sorted by off-balance sheet items. See captions in Table 3 for details.

Variables	Median of estimates: OBSI-sorted					Frequencies of rejection individual $t$ -test				
	Total Sample	Top	Q3	Q2	Bottom	Total Sample	Top	Q3	Q2	Bottom
Constant	0.007	0.000	0.009	0.018	0.005	42.18%	29.52%	42.86%	63.81%	32.71%
VIX	-0.002	-0.001	-0.002	-0.002	-0.002	96.21%	92.38%	99.05%	94.29%	99.07%
Market Return	0.060	0.026	0.065	0.145	-0.010	32.23%	20.95%	32.38%	56.19%	19.63%
$\Delta T$ -bill	-0.003	0.009	-0.006	-0.046	0.019	41.47%	33.33%	40.95%	54.29%	37.38%
Yield Slope	0.002	0.001	0.002	0.002	0.003	29.86%	20.95%	32.38%	18.10%	47.66%
Default Premium	-0.005	-0.002	-0.006	-0.009	-0.005	37.68%	21.90%	36.19%	56.19%	36.45%
NBER Recesions	-0.001	-0.005	-0.002	0.000	0.002	22.04%	27.62%	22.86%	29.52%	8.41%
Financial Recession	-0.034	-0.028	-0.038	-0.038	-0.028	88.63%	99.05%	91.43%	86.67%	77.57%
$X_{t,1}^-$	0.020	0.028	0.020	0.017	0.015	74.41%	87.62%	79.05%	73.33%	57.94%
$X_{t,2}^-$	0.017	0.025	0.020	0.014	0.011	42.18%	65.71%	44.76%	34.29%	24.30%
$X_{t,3}^-$	0.018	0.025	0.018	0.017	0.011	28.91%	46.67%	31.43%	20.95%	16.82%
$X_{t,4}^-$	0.024	0.033	0.022	0.013	0.018	17.30%	26.67%	16.19%	15.24%	11.21%
$X_{t,5}^-$	0.027	0.046	0.026	0.011	0.011	17.30%	19.05%	15.24%	17.14%	17.76%
$X_{t,1}^+$	0.002	0.012	-0.005	-0.006	0.000	13.27%	10.48%	12.38%	18.10%	12.15%
$X_{t,2}^+$	0.003	0.005	0.004	-0.003	0.003	10.66%	12.38%	5.71%	14.29%	10.28%
$X_{t,3}^+$	0.007	0.012	0.006	0.002	0.009	13.27%	19.05%	13.33%	14.29%	6.54%
$X_{t,4}^+$	0.006	0.010	0.005	0.002	0.005	21.80%	38.10%	20.00%	17.14%	12.15%
$X_{t,5}^+$	0.003	0.005	0.004	0.001	0.002	28.91%	40.95%	23.81%	28.57%	22.43%
Pseudo- $\mathcal{R}^2$	33.1%	36.2%	33.2%	34.5%	28.0%	-	-	-	-	-

**Table 6.** Frequencies of rejection at the 95% confidence level of different composite hypotheses characterizing the shape of the MRP of the system given the banks in the total sample and on the size-sorted quartiles of total assets, short-term wholesale funding and off-balance sheet items.

Composite hypotheses	Total Sample	Sorted by: Total assets			
		Top	Q3	Q2	Bottom
$H_{0,Ind} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = 0$	93.84%	100.00%	97.14%	94.29%	84.11%
$H_{0,Sym} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = \delta_i(\tau)$	78.67%	91.43%	80.00%	77.14%	66.36%
$H_{0,Asym} : \delta_{ij}^-(\tau) = \delta_i^-(\tau), \delta_{ij}^+(\tau) = \delta_i^+(\tau)$	45.02%	48.57%	41.90%	47.62%	42.06%
$H_{0,EqLoss} : \delta_{ij}^-(\tau) = \delta_i^-(\tau)$	29.62%	20.00%	28.57%	37.14%	32.71%
$H_{0,EqGain} : \delta_{ij}^+(\tau) = \delta_i^+(\tau)$	27.73%	40.95%	25.71%	19.05%	25.23%
		Sorted by: STWSF			
	Total Sample	Top	Q3	Q2	Bottom
$H_{0,Ind} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = 0$	93.84%	100.00%	99.05%	91.43%	85.05%
$H_{0,Sym} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = \delta_i(\tau)$	78.67%	90.48%	85.71%	69.52%	69.16%
$H_{0,Asym} : \delta_{ij}^-(\tau) = \delta_i^-(\tau), \delta_{ij}^+(\tau) = \delta_i^+(\tau)$	45.02%	45.71%	46.67%	41.90%	45.79%
$H_{0,EqLoss} : \delta_{ij}^-(\tau) = \delta_i^-(\tau)$	29.62%	20.95%	28.57%	34.29%	34.58%
$H_{0,EqGain} : \delta_{ij}^+(\tau) = \delta_i^+(\tau)$	27.73%	36.19%	28.57%	19.05%	27.10%
		Sorted by: OBSI			
	Total Sample	Top	Q3	Q2	Bottom
$H_{0,Ind} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = 0$	93.84%	100.00%	93.33%	93.33%	88.79%
$H_{0,Sym} : \delta_{ij}^-(\tau) = \delta_{ij}^+(\tau) = \delta_i(\tau)$	78.67%	91.43%	76.19%	79.05%	68.22%
$H_{0,Asym} : \delta_{ij}^-(\tau) = \delta_i^-(\tau), \delta_{ij}^+(\tau) = \delta_i^+(\tau)$	45.02%	45.71%	49.52%	50.48%	34.58%
$H_{0,EqLoss} : \delta_{ij}^-(\tau) = \delta_i^-(\tau)$	29.62%	23.81%	33.33%	36.19%	25.23%
$H_{0,EqGain} : \delta_{ij}^+(\tau) = \delta_i^+(\tau)$	27.73%	36.19%	21.90%	28.57%	24.30%

**Table 7.** Medians of parameter estimates and average frequencies of rejection of t-statistics of individual significance on banks grouped by the median of total assets,  $\text{Med}(\text{TA})$ , in billions over the period. The rows  $\text{H}_{0,EqLoss}$  and  $\text{H}_{0,EqGain}$  show the rejection frequencies of  $\text{H}_{0,EqLoss} : \delta_{ij}^-(\tau) = \delta_{ij}^-(\tau)$  and  $\text{H}_{0,EqGain} : \delta_{ij}^+(\tau) = \delta_{ij}^+(\tau)$  on these subsamples, respectively.

Variables	$\text{Med}(\text{TA}) \geq 250$			$100 \leq \text{Med}(\text{TA}) < 250$			$50 \leq \text{Med}(\text{TA}) < 100$			$\text{Med}(\text{TA}) < 50$		
	Median Estimates	Freq. rej. $t$ -stat	$t$ -stat	Median Estimates	Freq. rej. $t$ -stat	$t$ -stat	Median Estimates	Freq. rej. $t$ -stat	$t$ -stat	Median Estimates	Freq. rej. $t$ -stat	$t$ -stat
Constant	0.002	0.00%		0.001	16.67%		-0.003	0.00%		0.009	43.92%	
VIX	-0.001	83.33%		-0.001	100.00%		-0.001	100.00%		-0.002	96.28%	
Market Return	-0.069	0.00%		0.037	33.33%		0.023	14.29%		0.063	33.00%	
$\Delta T$ -bill	-0.012	16.67%		0.007	83.33%		0.002	0.00%		-0.003	41.69%	
Yield Slope	0.000	0.00%		0.000	16.67%		0.000	0.00%		0.002	31.02%	
Default Premium	0.000	33.33%		-0.001	33.33%		0.000	100.00%		-0.006	38.46%	
NBER Recessions	-0.011	66.67%		-0.014	66.67%		-0.010	100.00%		0.000	20.35%	
Financial Recession	-0.022	100.00%		-0.006	100.00%		-0.046	100.00%		-0.034	88.09%	
$X_{t,1}^-$	0.034	100.00%		0.032	100.00%		0.035	85.71%		0.020	73.20%	
$X_{t,2}^-$	0.039	100.00%		0.040	66.67%		0.030	71.43%		0.016	39.95%	
$X_{t,3}^-$	0.043	100.00%		0.044	66.67%		0.036	28.57%		0.017	26.30%	
$X_{t,4}^-$	0.049	83.33%		0.043	33.33%		0.041	0.00%		0.021	15.14%	
$X_{t,5}^-$	0.120	50.00%		0.023	16.67%		0.061	0.00%		0.024	16.63%	
$X_{t,1}^+$	-0.020	0.00%		0.014	0.00%		0.018	57.14%		0.002	13.90%	
$X_{t,2}^+$	0.019	0.00%		0.001	16.67%		-0.001	71.43%		0.003	10.92%	
$X_{t,3}^+$	0.010	16.67%		0.016	50.00%		0.009	71.43%		0.007	11.91%	
$X_{t,4}^+$	0.020	83.33%		0.015	33.33%		0.015	71.43%		0.006	19.85%	
$X_{t,5}^+$	0.009	83.33%		0.006	50.00%		0.007	71.43%		0.003	27.05%	
Pseudo- $\mathcal{R}^2$	48.3%	-		43.4%	-		43.8%	-		32.7%	-	
$\text{H}_{0,EqLoss}$	66.7%	-		16.7%	-		14.3%	-		29.8%	-	
$\text{H}_{0,EqGain}$	33.3%	-		83.3%	-		42.9%	-		26.8%	-	